Printed Page:-04 **Subject Code:- AMIAS0103**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech (Integrated)

SEM: I - THEORY EXAMINATION - (2023 - 2024)

Subject: Engineering Mathematics I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. If the rank of A is 2, then rank of A' is (CO1) 1

(a) 3

(b) 2

(c) 8

(d) 16

1-b. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then the eigen values of A^2 are (CO 1) 1

(a) 1, 7, 9

(b) 1, 4, 9

(c) 1, 1, 2

(d) 3, 6, 9

1-c. $\text{If } u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}, \text{then the value of } x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} \text{ is}$ (CO2) 1

(a) 0

(b) u

(c) 4

(d) 1

1-d. The asymptotes parallel to the y-axis of the curve $x^2y^2 - a^2(x^2 + y^2) = 0$ is 1

(CO2)

(a) 2a

(b) $x = a$

(c) $x = 3a$

(d) None of these

1-e. Maclaurin's series for f(x) is (CO3) 1

(a) $f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+.....$

(b) $f(x)+\frac{x}{1!}f'(x)+\frac{x^2}{2!}f''(x)+\frac{x^3}{3!}f'''(x)+.....$

(c) $f(0)+\frac{x}{1}f'(0)+\frac{x^2}{2}f''(0)+\frac{x^3}{3}f'''(0)+.....$

(d) $f(x)+\frac{1}{1!}f'(x)+\frac{1}{2!}f''(x)+\frac{1}{3!}f'''(x)+.....$

1-f. An error of 2% is made in measuring length and breadth then the percentage error in the area of the rectangle is (CO3) 1

(a) 6

(b) 4

(c) 8

(d) 16

1-g. The value of $\Gamma n\,\Gamma(1-n)$ is (CO4) 1

(a) $\frac{\pi}{\sin n\pi}$

(b) $\frac{2\pi}{\sin n\pi}$

(c) $\frac{\pi}{\cos n\pi}$

(d) None of these

1-h. 1

Value of the integral $\int_0^1 x^5(1-x^3)^3\,dx$ is CO 4

(a) 1/50

(b) 1/60

(c) 1/30

(d) -1/30

1-i. A got 37.5 % marks less than B, then by what percent the marks of B is more than the marks of A? (CO5) 1

(a) 60%

(b) 37.5%

(c) 27.27%

(d) 40%

1-j. If blue is coded as green, green is coded as white and white is code as black, and then what will be the code for the colour of grass? (CO5) 1

(a) White
(b) Green
(c) Black
(d) None of These

2. Attempt all parts:-

2.a. The eigen values of A are 2, 3, 1, then find the eigen values of $A^{-1} + A^2$.(CO1) 2

2.b. State the Leibnitz theorem. (CO2) 2

2.c. Find the minimum value of $\sqrt{x^2+y^2}$. (CO3) 2

2.d. Evaluate the value of $\frac{\beta(m+1,n)}{\beta(m,n)}$. (CO4) 2

2.e. Find the missing terms of 2, 4, 16, 512, ? (CO5) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. If $u = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ (CO 1) 6

3-b. Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary transformation. (CO1) 6

3-c. If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial}{\partial} = -3\tan u$. (CO2) 6

3-d. If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt (CO2) 6

3.e. Expand $f(x,y) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's series expansion about the point (1,2) upto three terms. (CO 3) 6

3.f. Evaluate $\int_0^1 \int_{y^2}^{y} (1+xy^2)\,dx\,dy$. (CO 4) 6

3.g. The marked price of a pencil is 35% more than its cost price. What maximum discount percentage can be offered by the shopkeeper to sell his pencil at no profit or no loss? (CO5) 6

SECTION-C 50

4. Answer any one of the following:-

4-a. Determine the value of λ and μ so that the equations 10

$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = \mu$

have (i) no solution (ii) a unique solution (iii) infinite many solutions. (CO1)

4-b. Verify Caley-Hamilton Theorem for the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. (CO1) 10

5. Answer any one of the following:-

5-a. If $y = \sin(a \sin^{-1} x)$, then find $y_n(0)$. (CO2) 10

5-b. Trace the curve $y^2(2a - x) = x^3$. (CO2) 10

6. Answer any one of the following:-

6-a. If $u^3+v^3+w^3 = x+y+z$, $u^2+v^2+w^2 = x^3+y^3+z^3$ and $u+v+w = x^2+y^2+z^2$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$. (CO3) 10

6-b. A rectangular box closed at the top is of given volume, what must be the dimensions so that the surface area is minimum. (CO3) 10

7. Answer any one of the following:-

7-a. Evaluate $\int\int\int x^2yz \, dx \, dy \, dz$, throughout the volume bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (CO 4) 10

7-b. By changing the order of integration evaluate $\int_0^2 \int_{x^2/4}^{3-x} xy \, dy dx$. (CO4) 10

8. Answer any one of the following:-

8-a. (i) If the radius of the cylinder increases by 10 % and the height increases by 20%. Then, what is the change in the volume of the cylinder ?
(ii) The average age of eight teachers in a school is 40 years. A teacher among them died at the age of 55 years whereas another teacher whose age was 39 years joins them. The average age of the teachers in the school now is (in years)
(iii) A machine is sold for Rs5060 at a gain of 10%. What would have been the gain or loss % if it had been sold for Rs 4370?
(CO5) 10

8-b. (i) The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. Find the present age of husband? 10

(ii) A tradesman sold an article at a loss of 20%. If the selling price had been increase by Rs. 100, there would have been a gain of 5%. Find the cost price of the article?

(iii) Find the missing term
122, 62, 32, ?, 9.5, 5.75 . (CO5)